Error Analysis

Finding the root-mean-square (rms) error

Consider that you have measured the length of a rod 10 times, you are asked to find the average length along with its error.

N	L (m)
1	2.52
2	2.69
3	2.46
4	2.58
5	2.39
6	2.41
7	2.62
8	2.66
9	2.67
10	2.56

$\bar{L} = 2.556 m$

$\alpha = 0.034 m = \frac{S.D}{\sqrt{N}}$

$\rightarrow L = (2.56 \pm 0.03)\ m$

To attain the results using the calculator apply the following

Casio fx-570: mode $\rightarrow$ 3 $\rightarrow$ 1 (fill table) $\rightarrow$ AC $\rightarrow$ shift 1 $\rightarrow$ 4 $\rightarrow$ $\begin{cases} 2 - \bar{x} \\ 4 - S.D \\ \alpha = \frac{S.D}{\sqrt{N}} \end{cases}$

Casio fx-95: mode $\rightarrow$ 2 $\rightarrow$ (value M+) $\rightarrow$ shift 2 $\rightarrow$ $\begin{cases} 1 - \bar{x} \\ 3 \rightarrow S.D \rightarrow \alpha = \frac{S.D}{\sqrt{N}} \end{cases}$

Propagation of error

Consider an experiment where you measured the period of a pendulum and from it you are asked to find the gravitational acceleration g along with its error.

The period of the pendulum is given by! $T = 2\pi\sqrt{\frac{l}{g}}$

where $T = (1.94 \pm 0.15)s$; $l = (0.93 \pm 0.12) m$

$$g = \frac{4\pi^2 l}{T^2} = \frac{4\pi^2 \times 0.93}{1.94^2} = 9.7553 \ m/s^2$$

$$\alpha_g = \sqrt{\left(\frac{\partial g}{\partial l}\alpha_l\right)^2 + \left(\frac{\partial g}{\partial T}\alpha_T\right)^2}$$

* $\frac{\partial g}{\partial l}\alpha_l = \frac{4\pi^2}{T^2}\alpha_l = \frac{4\pi^2}{1.94^2} \times 0.12 = 1.26 \ m/s^2$

* $\frac{\partial g}{\partial T}\alpha_T = -\frac{8\pi^2 l}{T^3}\alpha_T = -1.51 \ m/s^2$

$\Rightarrow \alpha_g = 1.97 \ m/s^2$

$\Rightarrow g = (10.0 \pm 2.0) \ m/s^2$

Linear Regression

Assume you have an experimental setup were you vary one factor and measure another

x	y	e_i	e_i^2
60	3.1	0.205	0.0420
61	3.6	-0.108	0.0117
62	3.8	-0.120	0.0144
63	4	-0.132	0.0174
65	4.1	0.144	0.0206
$\Sigma x = 311$			$\Sigma e_i^2 = 0.1061$
$\Sigma x^2 = 19359$			

* Write ~~down~~ the relation between the given data

* Choose x & y ~~such that you~~ from the data such that you get a linear relation in accordance to the formula above (write them down e.g let column 1 be x and column 2 be y)

slope $= B = 0.1878$ unit (y/x)

intercept $= A = -7.9635$ (unit of y)

$\Delta = N\Sigma x^2 - (\Sigma x)^2 = 74$

$e_i = Bx_i + A - y_i$

$\alpha_{slope} = \sqrt{\frac{N}{N-2} \frac{\Sigma e_i^2}{\Delta}} = 0.048$ unit

$\Rightarrow$ slope $= (0.19 \pm 0.05)$ unit

~~How~~ to get the values from the calculator:

fx - 570

mode $\to 3 \to 2$ (fill table) $\to$ AC $\to$ shift $1 \to \begin{cases} 3 \to \begin{cases} 1 \to \Sigma x^2 \\ 2 \to \Sigma x \end{cases} \\ 5 \to \begin{cases} 2 - B \\ 1 - A \end{cases} \end{cases}$

fx - 95: mode $\to 3 \to 1 \to$ x,y M+ $\to$ shift $1 \to \begin{cases} 1 \to \Sigma x^2 \\ 2 \to \Sigma x \end{cases}$

shift 2 (go to end of menu) $\to \begin{cases} 2 - B \\ 1 - A \end{cases}$

Significant figures

* you round the error to the 1^{st} significant figure (the 2^{nd} when the exception applies), then you match the number of ~~place~~ decimal places in the value to that of the error

* if the last significant figure happens to be zero after rounding, the corresponding decimal place in the value must be zero

* whenever scientific notation is involved (powers of 10) you factor the ~~ex~~ power of 10 of the value ~~then~~ and you write the remaining for the error in decimal form then round for significant figures

Comparing

In the example of the pendulum, we got the gravitational acceleration: $g = (10.0 \pm 2.0)\ m/s^2$
we are asked to compare this to the theoretical value of $9.81\ m/s^2$. To ~~this~~ this, we check if the theoretical value belongs to the interval $[g_{exp}-\alpha, g+\alpha]$

$$g_{theo} \overset{?}{\in} [g_{exp}-\alpha, g_{exp}+\alpha]$$

$$9.81 \in [8.0, 12.0]$$

we comment that our experiment is accurate
However, should the theoretical value not belong to the interval, you try for 2α.

* No matter how obvious it may seem, you must always try α first

* to comment about precision, you check the ratio of the error to the calculated value, if it is below 10% then your measurement is precise else it is not

Now suppose you measure the same value using 2 different methods and are asked to compare, so in the lab you measure the following: $x_1 \pm \alpha_1$
$x_2 \pm \alpha_2$

to compare, you check that the intersection is not an ~~different~~ empty set.

i.e $[x_1-\alpha_1, x_1+\alpha_1] \cap [x_2-\alpha_2, x_2+\alpha_2] \neq \emptyset$

Should the intersection be $\emptyset$, you multiply both errors by 2 and try again. As for comments about accuracy, the same applies, as for precision, you check individual measurements.

What You should be aware of during calculations

1) check units of given data, either convert everything to S.I then calculate or calculate everything then convert in the end

2) should you finish and notice any missing power of 10, you probably missed a conversion. If the value itself is way off you might have done a mistake in either calculation or measurement.

* Rules

- Always bring Graph paper
- Always come prepared
- no cell phones
- no cheating
- bring calculator, ruler, pens .....